Risk and Return Relations for Treasury Bonds

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Abstract

Where the literature that examines the relation between the expected excess return and conditional volatility of common stocks can be characterized as a search for a positive relation, we find that a positive relation between expected return and conditional volatility is conspicuous for bonds. Contrary to findings for stocks, our inferences regarding the sign of the risk-return relation for bonds are robust to changes in the empirical specification of conditional volatility. Similar to findings for stocks, we find evidence of instability in the short-horizon relation between bond risk and return which suggests that the reward to bond volatility varies over time.

Key words: Treasury bond, excess return, conditional volatility, risk premiums

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1. Introduction

We examine the intertemporal relation between the conditional expected return and conditional volatility of U.S. Treasury bonds, a relation for which there is little investigation in the existing literature. Our goal is to provide evidence on bond risk and return comparable to the extensive evidence in the stock market literature. Thus, we use the existing literature on the stock market as a guide in documenting both the long-run relation between expected return and volatility and the stability of that relation.

Despite the current lack of attention, the Treasury bond risk-return relation is of more than passing interest, having implications for both portfolio selection and empirical asset pricing. For portfolio selection, a central concern is whether or not a bond provides a return commensurate with the risk of holding the bond. A related concern is whether bonds can serve as an intertemporal hedging asset as suggested by Merton (1980). An examination of the time series of bond risk and return can shed light on both issues. In the asset pricing literature, the primary focus for bonds has been on the modeling of the term structure of yields. Recent examples are the affine term structure models of Dai and Singleton (2000) and Duffee (2002). There is a danger that such models overfit yields, weakening their ability to explain other aspects of bond returns. Empirical evidence on the bond risk-return relation provides benchmarks for specification checks designed to test whether or not models designed to fit bond yields adequately describe aspects of bond holding period returns.1

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1 Early evidence provided by Pilotte and Sterbenz (2006) suggests that the essentially affine models of Duffee (2002), which nest the completely affine models of Dai and Singleton (2000) fail to capture important aspects of holding period risk and return.
The focus of our empirical analysis is on applying parametric methods commonly used in the stock literature, to examine monthly excess returns for a variety of bond maturities during 1961 - 2002. Because of the paucity of existing evidence on the risk-return relation for bonds relative to stocks, limiting our study to parametric methods and monthly returns keeps the scope of the analysis manageable and provides a reasonably rich baseline for future study, while supplying many results comparable to key findings in the existing stock literature. For the same reasons, we reserve the use of regime switching models for subsequent work.

For our full sample, we find a significant positive relation between bond excess returns and conditional variance for most maturities. Contrary to the existing findings for stocks, neither the sign nor the significance of the risk-return relation is very sensitive to the choice of estimation method. Specifically, the results are similar whether the conditional variance is modeled using only financial conditioning variables, a simple generalized autoregressive conditional heteroskedasticity in mean (GARCH-M) model, a GARCH-M model that incorporates financial conditioning variables in the estimation of the conditional variance, or GARCH-M models that allow for asymmetries in the conditional variance equation. While all of our models provide evidence consistent with a positive risk-return relation, the strongest results are for the model that incorporates both financial conditioning information and GARCH effects in estimating the conditional variance. Thus, combining alternative methods of estimating the conditional variance reinforces inferences regarding the sign of the risk-return relation. This contrasts with the stock literature, where combining alternative methods often changes inferences.

For all of our models, estimates of the risk-return relation display a distinct pattern in that they are very high for short-term bonds and low for long-term bonds. The tendency for these estimates of the reward to volatility to decline with bond maturity confirms, with an alternative methodology, recent findings by Pilotte and Sterbenz (2006).

We also examine the linearity and stability of the relation between conditional mean and conditional variance. For each model of conditional variance and each bond maturity, regression
analysis indicates that financial conditioning information explains variation in bond excess returns that is not related to changes in the conditional variance. These results suggest that a time invariant linear model of the risk-return relation is misspecified and that the reward to bond volatility changes over time. This conclusion is consistent with prior evidence on the risk-return relation for common stocks.

An examination of rolling correlations between our best estimates of the conditional mean excess return and conditional variance shows substantial variation over time in the short-horizon relation between bond risk and return. The relation is often negative. For longer maturities, both the range of correlations and incidence of negative correlations are similar to those reported previously for common stocks. For shorter maturities the range is diminished somewhat; however, the rolling correlations for all bond maturities do tend to move together.

The remainder of this paper is organized as follows. Section 2 reviews related literature. Section 3 provides theoretical context. Section 4 describes the data. Section 5 presents our empirical model of conditional mean excess returns and tests of the stability of the process generating excess returns. Section 6 presents evidence regarding the relation of bond excess returns to their conditional volatilities using various methods to model conditional volatility. Section 7 evaluates the linearity and stability of the relation between the conditional mean and conditional variance. Section 8 concludes.

2. Related Literature

This section reviews the prior literature on the relation between the conditional expected return and volatility of stocks and bonds. The extensive stock literature provides a guide to executing and context for interpreting our extension of the limited bond literature. There is reason to expect an examination of bond returns to yield both similarities and differences to stock returns. In monthly data for 1959 – 1983 Campbell (1987) documents the fact that variables used to predict excess bond returns also predict excess stock returns, which suggests the possibility of
common variation in risk and return for the two asset classes. On the other hand, for the 1950 – 1999 period Reilly, Wright, and Chan (2000) find that return volatility is more stable for stocks than for bonds, the ratio of stock market to bond market volatility is not stable, and the correlation between bond and stock returns varies widely. Jones and Wilson (2004) find similar results for the period 1871 – 2000.

2.1. Stocks

The literature contains contradictory findings regarding the relation between the conditional mean and conditional volatility of monthly stock market returns. Results differ according to the methodology used to estimate the conditional volatility. For a simple GARCH-M model, French, Schwert, and Stambaugh (1987) report a positive, but only marginally significant, estimated coefficient on volatility in expected return regressions using monthly data. This result holds for both the conditional variance and conditional standard deviation. To the contrary, Glosten, Jagannathan, and Runkle (1993) find a negative relation in an asymmetric GARCH-M model that allows positive and negative returns to have different effects on the conditional variance. This relation becomes stronger when the conditional variance is modeled with both GARCH effects and predetermined conditioning variables. Campbell (1987) and Whitelaw (1994) report a negative correlation between the conditional mean and conditional volatility when both moments are modeled as functions of predetermined financial conditioning information. Harvey (2001) examines a variety of parametric and nonparametric methods for estimating conditional expectations. Harvey concludes that models that take into account financial conditioning information produce a significant negative relation between conditional mean and variance, while other approaches produce positive, insignificant coefficients.\footnote{French, Schwert, and Stambaugh (1987) find that the volatility coefficients are significantly positive for GARCH estimation using daily returns.} \footnote{Another approach is to use daily returns to model monthly volatility. French, Schwert, and Stambaugh (1987) use within-month daily returns to estimate monthly volatility and report a positive but insignificant relation between the conditional mean and volatility of stock returns. Ghysels, Santa-Clara, and Valkanov}
Two studies present evidence on the stability of the relation between the conditional mean and volatility of stock returns. Whitelaw (1994) estimates the conditional moments of stock returns using financial conditioning information and finds that the short-term correlation between fitted values of the conditional mean and conditional volatility varies substantially over time. He attributes this variation to the fact that volatility leads the expected return over the business cycle. Harvey (2001) statistically rejects a time invariant linear relation between conditional mean and conditional variance, and presents evidence that the ratio of the fitted moments displays a distinct business cycle pattern. Harvey’s findings are robust to changes in the methods used to estimate conditional means and variances.

2.2. Bonds

Two studies report direct evidence regarding the intertemporal relation between the conditional mean and conditional volatility of bond returns. Engle, Lilien, and Robins (1987) use an ARCH-M framework to estimate the relation between the conditional mean and conditional standard deviation of monthly excess holding period returns on two-month Treasury bills and twenty-year AAA rated corporate bonds. They find positive coefficient estimates on volatility in the expected return regressions for both return series. The coefficient for the two-month bill is significant at the 0.01 level, while that for corporate bonds is significant at the 0.10 level. Campbell (1987) estimates the conditional mean and conditional variance of monthly excess returns on two-month Treasury bills, six-month treasury bills, and a portfolio of five-to-ten-year Treasury bonds, where both moments are modeled as functions of financial conditioning variables. Campbell reports correlations between the fitted moments of 0.625 for the two-month bill, 0.835 for the six-month bill, and 0.029 for the long-term bond portfolio. While the evidence reported in these studies is limited in terms of the bond maturities examined, the two studies are (2005) introduce an estimator that forecasts monthly variance as a weighted average of past daily squared returns, where the weights on lagged daily squared returns are parameterized with a flexible functional form. This approach produces a significant positive relation between the conditional mean and conditional variance of the aggregate stock market return.
consistent in reporting a strong positive relation between risk and return for short-term bills and a weak positive relation for long-term bonds.

Less direct evidence that a bond’s excess return may be related to its own volatility is provided by Fama (1976) and Klemkosky and Pilotte (1992). Viewed together, these studies document positive relations between excess returns and the volatility of the one-month bill rate for a variety of bill and bond maturities. In single state variable models of the term structure of interest rates, such as Cox, Ingersoll, and Ross (1985), short-rate volatility is perfectly correlated with each bond’s own total volatility. Interpreted in the context of these models, the documented positive relation between bond excess returns and short-rate volatility implies a positive relation between excess returns and own volatility. However, it is now widely recognized that at least three state variables are required to adequately model the term structure, so this interpretation is an overstatement.4

No study presents a direct test of the stability of the relation between conditional expected excess returns and volatilities for bonds; however, one study reports results that are suggestive. Pilotte and Sterbenz (2006) examine business cycle patterns in the conditional mean, conditional volatility, and ratio of conditional mean to conditional volatility (the Sharpe ratio) for Treasury bills, Treasury bonds, and stocks. They model excess returns as functions of financial conditioning information, using GARCH estimation of the excess return regressions to produce estimates of the conditional volatilities. For short-maturity bills and bonds, they find that both the conditional mean and volatility tend to peak at the same point in the business cycle. For long-maturity bonds and stocks the volatility tends to peak prior to the conditional mean; that is, long-term bond returns appear to share the time series patterns of risk and return that Whitelaw (1994) documents for stocks. These results suggest that a stable positive relation between risk and return is more likely for shorter than for longer maturity bonds.

4 Litterman and Scheinkman (1991) find that 96% of changes in bond values are captured by three factors that characterize the level, curvature, and steepness of the term structure.
3. Theoretical Context

We consider the intertemporal choice problem of a representative investor who maximizes the expectation of a time-separable utility function. In that case, assets can be priced as the conditional expected value of their payoff with a pricing operator, that is

\[ P_{i,t} = E_t[M_{t+1}(P_{i,t+1} + I_{i,t+1})], \tag{1} \]

where \( P_{i,t} \) is the nominal price of asset \( i \) at time \( t \), \( I_{i,t+1} \) is the asset’s nominal income at \( t+1 \), and \( M_{t+1} \) is the pricing operator. The pricing operator is the marginal rate of substitution, defined as \( M_{t+1} ≡ \beta U'(C_{t+1})/U'(C_t) \), where \( \beta \) is the time preference parameter and \( U(C_t) \) defines utility as a concave function of time \( t \) consumption, \( C_t \). Defining the gross return as

\[ R_{i,t+1} = (P_{i,t+1} + I_{i,t+1})/P_{i,t}, \tag{2} \]

equation (1) can be rewritten in terms of asset returns

\[ 1 = E_t[M_{t+1} R_{i,t+1}]. \tag{2} \]

The one-period risk-free rate of return, \( R_{f,t} \), is known at time \( t \), so that

\[ R_{f,t} = E_t[M_{t+1}]^{-1}. \tag{3} \]

The price of a \( \tau \)-period-to-maturity risk-free discount bond that pays $1 at maturity is

\[ P_{\tau,t} = E_t[M_{t+1,t+\tau}], \tag{4} \]

where \( E_t[M_{t+1,t+\tau}] = E_t[M_{t+1}M_{t+2} \cdots M_{t+\tau}] \), and the one-period return on the \( \tau \)-period discount bond is:

\[ R_{\tau,t+1} = \frac{P_{\tau,t+1}}{P_{\tau,t}} = \frac{E_t[M_{t+2,t+\tau}]}{E_t[M_{t+1,t+\tau}]} \tag{5} \]

Equation (5) tells us that the holding period return on a bond is a function of changes in expectations of future values of the marginal rate of substitution (MRS) over the bond’s life.

Thus, business cycle patterns in bond returns and their volatilities are likely to emerge as

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5 Equation (1) can also be derived from the absence of arbitrage, without assuming that investors maximize well-behaved utility functions. Chapter 4 of Cochrane (2001) contains discussion of the minimum requirements for equation (1) to hold.
investors adjust their expectations of future MRS in light of shocks to their expected consumption stream. Equation (5) is consistent with potentially complex, nonlinear relations between risk and return.

Expanding equation (2) and rearranging, the one-period risk premium to holding any asset $i$ is

$$E_t[R_{i,t+1}] - R_{f,t} = -R_{f,t} \text{Cov}_t[M_{t+1}, R_{i,t+1}],$$

(6)

where $\text{Cov}_t$ is the conditional covariance at time $t$. Equation (6) introduces intertemporal hedging issues. According to equation (6), an asset will earn a positive risk premium if its return is inversely related to the MRS, that is, if the asset return is high when the marginal utility of consumption is low and low when marginal utility is high. However, a negative risk premium is indicated for hedging assets, that is, assets that have high payoffs when the marginal utility of consumption is high and low payoffs when marginal utility is low. Equation (6) suggests business cycle patterns in expected excess returns, since the MRS should vary inversely with expected consumption growth.

Substituting equation (5) into equation (6) produces the following expression for the excess return to the $\tau$-period bond:

$$E_t[R_{\tau,t+\tau}] - R_{f,t} = -R_{f,t} \text{Cov}_t \left[ M_{t+1}, \frac{E_{t+\tau}[M_{t+2,t+\tau}]}{E_t[M_{t+1,t+\tau}]} \right].$$

(7)

Equation (7) demonstrates that the ex ante risk premium on a bond reflects the expected time series properties of the MRS during the bond’s maturity. Thus, bonds of adjacent maturities are likely to have similar return characteristics. Characteristics of short and long maturity bonds could be very different.

In empirical studies the spread between a bond’s yield and the risk-free rate is often used as a conditioning variable for predicting the excess return. Equation (4) implies that the gross yield on a $\tau$-period discount bond is
A comparison of equations (7) and (8) shows why the term spread has been found to be a useful conditioning variable for predicting bond excess returns.

Expanding equation (6) provides the relation of the ex ante risk premium to an asset’s own volatility

\[
E_t[R_{i,t+1}] - R_{f,t} = -R_{f,t} vol_i[M_{t+1}] vol_t[R_{i,t+1}, corr_t[M_{t+1}, R_{i,t+1}]].
\]

where \( vol_i \) is the conditional standard deviation and \( corr_t \) is the conditional correlation.

Equation (9) shows that the volatility of an asset’s return is not necessarily a good proxy for priced risk. Only when the correlation between an asset’s return and the MRS is equal to minus one is its conditionally expected risk premium perfectly positively correlated with its conditional volatility. A weak positive relation between the conditionally expected risk premium and volatility occurs when \(-1 < corr_t < 0\). A weak negative relation occurs when \(0 < corr_t < 1\). For an asset that represents a perfect hedge, with \( corr_t = 1\), there will be a perfect negative correlation between the conditionally expected premium and volatility.

Several conclusions can be drawn from the general model of asset pricing presented in the section. First, for any asset, potentially complex and time-varying patterns of risk and return are possible. Second, for bonds of different maturities, patterns of risk and return are likely to differ. The difference is likely small for adjacent maturity bonds and potentially large for short versus long-term bonds, because the holding period return for each bond depends on changes during the holding period in expected values of the MRS over the remaining life of the bond, as shown by equation (5). Equation (7) shows that this risk is priced ex ante. Third, to the extent that stock and bond returns are less than perfectly correlated, patterns of risk and return will differ for bonds versus stocks as shown in equation (9). Fourth, equation (9) indicates that some similarities in the behavior of returns on different assets are likely, because all asset returns reflect the ex ante volatility of the MRS. Our goal for the remainder of this paper is to document the
potentially rich patterns of bond risk and return and to compare them to the existing evidence for common stocks.

4. Data and Descriptive Statistics

Data are from the Center for Research in Security Prices (CRSP). Returns are one-month holding period returns. Returns and yields on one-month and three-month to maturity Treasury bills are from the Fama Treasury Bill Term Structure Files. Returns on five Treasury bond portfolios are from the Fama Maturity Portfolios Returns File with bonds grouped by maturities in one year intervals. Thus, the bond portfolios consist of bonds with maturities of less than 1, 1-2, 2-3, 3-4, and 4-5 years. Only non-callable, non-flower bonds and notes are included in the portfolios. Yields that correspond to the portfolio returns are from the Fama-Bliss Discount Bonds File. Each yield is for the discount bond at the upper bound of maturity allowed in a portfolio. There are gaps in the return history for bond portfolios of maturities greater than 5 years, so we use returns and yields on the ten-year and twenty-year constant maturity bonds from the CRSP Fixed Term Indices Files to represent longer maturity bonds. Where possible, CRSP uses a non-callable, non-flower bond in constructing the Fixed Term Indices Files, but in some cases flower bonds are used. The sample period is January 1961 to December 2002. We start with January 1961, because there are often substantial gaps in prior months between the desired and available maturities for the ten- and twenty-year constant maturity bonds. Eight excess return series are calculated by subtracting the return to the one-month bill from the holding period returns on the three-month bill, each of the five bond portfolios, and the ten- and twenty-year constant maturity bonds.

We report descriptive statistics for the excess return series in Panel A of Table 1. Both the mean and standard deviation of monthly excess returns tend to increase with maturity.

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6 We use the twenty-year and not the thirty-year bond from the Fixed Term Indices File because there are several years where both series are based on the same bond and the gap between actual and desired maturity is generally smaller for the twenty-year bond.
Although the means increase with maturity, standard deviations rise more sharply. These results are consistent with Pilotte and Sterbenz (2006), who find that bond Sharpe ratios decline with maturity.

The Jacque-Bera (JB) statistics, a goodness-of-fit test of the departure of the distribution of a data series from the normal, reject normality at the 0.01 level for each excess return series. An examination of the skewness and kurtosis of the excess return series indicates that the rejection of normality is due predominately to excess kurtosis relative to the normal distribution. The Q(12) statistics reject the null hypothesis of no autocorrelation in the first 12 lags at the 0.05 or 0.01 level for seven of the eight series. Reported autocorrelations indicate that these rejections are due mostly to positive first order autocorrelation in the excess returns.

To examine aspects of the volatility of excess returns, we report descriptive statistics for squared excess returns in Panel B of Table 1. Panel B shows that both the mean and standard deviation of squared excess returns increase with maturity. The Q(12) statistics and autocorrelations reported in Panel B indicate substantial positive autocorrelation in squared excess returns that is more persistent than the positive autocorrelation in excess returns. These statistics suggest the existence of autoregressive conditional heteroskedasticity in each excess return series.

5. Excess Return Model and Model Evaluation

In this section we present our empirical model of conditional mean excess returns and carry out diagnostic tests to evaluate the stability of the model. The residuals of this model are used in a later section of this paper to model conditional volatility using predetermined financial conditioning information as instrumental variables.

5.1. Estimating Conditional Mean Excess Returns

In order to estimate the conditional volatility of a bond’s excess returns, it is useful to isolate the predictable and the unpredictable components of those returns. To do so, we model
the conditional mean excess return by regressing excess returns for each bond and bond portfolio on predetermined conditioning variables. An obvious choice for a conditioning variable is a bond’s own yield spread, defined as the beginning of period difference between the bond’s yield to maturity and the one-month T-bill rate. The yield spread has been shown to have predictive power for bond excess returns in prior studies by Campbell (1987), Fama (1990), and Pilotte and Sterbenz (2006). Fama (1990) shows that the yield spread contains the market’s estimate of the ex ante risk premium and should reflect variation in that premium. Fama’s inference is supported by our equations (7) and (8). Following Campbell (1987) and Pilotte and Sterbenz (2006) we also use the one-month bill rate as a conditioning variable. Pilotte and Sterbenz (2006) note that including the one-month rate adjusts the risk premium estimate in the yield spread for mean reversion in the one-month rate. Based on the positive first order autocorrelations in excess returns reported in Table 1, we also include the one-month lag of each bond’s excess return as a conditioning variable. Thus, our model of excess returns is:

\[ R_{\tau,t+1} - R_{f,t} = \alpha_{\tau,0} + \alpha_{\tau,1} R_{f,t} + \alpha_{\tau,2} (Y_{\tau,t} - R_{f,t}) + \alpha_{\tau,3} (R_{\tau,t} - R_{f,t-1}) + \varepsilon_{\tau,t+1} \]  

(10)

where \( t \) subscripts denote when a variable is observed, \( R_{\tau,t+1} \) is the uncertain return from holding from time \( t \) to \( t+1 \) a bond of maturity \( \tau \), \( R_{f,t} \) is the risk-free return known at time \( t \) and earned by holding a one-month bill from \( t \) to \( t+1 \), \( Y_{\tau,t} \) is the yield-to-maturity observed at time \( t \) on a bond of maturity \( \tau \), and \( \varepsilon_{\tau,t+1} \) is the error term.

The results of ordinary least squares estimation of regression equation (10) are reported in Table 2. The standard errors are adjusted for autocorrelation and heteroskedasticity. The one-month rate is significant (0.05 level) in explaining only the excess return to the three-month bill. The yield spread is significant at the 0.01 level for seven of the eight maturities and at the 0.05 level for the remaining maturity bond. The lagged excess return is significant at the 0.01 level for five bond maturities, at the 0.05 level for one maturity, and at the 0.10 level for one maturity. The regression R-square ranges from a low of 0.03 for the twenty-year bond to a high of 0.25 for
the three-month bill. These results support the conclusion that there is predictable variation in
bond excess returns for all maturities.

Table 2 also contains test statistics that examine aspects of the regression errors. The JB
statistics reject normality of the residuals at the 0.01 level for every regression. The White
statistics reject the null hypothesis of no heteroskedasticity at the 0.01 level for every regression.
The Breusch-Godfrey Lagrange Multiplier statistics reject the null hypothesis of no serial
correlation at the 0.01 level in five regressions and at the 0.10 level in one regression. Engle’s
Lagrange Multiplier ARCH statistics reject the null hypothesis of no autoregressive conditional
heteroskedasticity in the residuals at the 0.01 level in every regression. In brief, the regression
residuals are non-normally distributed, heteroskedastic, autocorrelated, and show strong evidence
of ARCH effects. These aspects of shocks to bond excess returns are considered in estimating the
models of the risk-return relation that appear later in this paper.

5.2. Evaluation of Excess Return Model

Klemkosky and Pilotte (1992) present evidence of shifts in the stochastic process that
generates Treasury bond risk premiums around October 1979 and October 1982 changes in
monetary policy. Thus, we conduct a variety of diagnostic tests to check the specification of our
model of excess returns.\footnote{Klemkosky and Pilotte (1992) reject the stability of a model of the relation between bond excess returns and short-rate volatility.}

Our first set of diagnostic tests is based on recursive least squares estimation of equation
(10) for each bond maturity. This procedure involves first estimating the regression for a bond
using the first four months of data. Data are then added one month at a time, using all data
available up to and including each month \( t \) to estimate the regression coefficients and residual for
month \( t \). This produces time series of coefficients and residuals that span the full sample period.

We examined plots against time of the recursive coefficients and two standard error
bands around the coefficients for each bond maturity. These plots suggest that the regression
coefficients are stable over time. We also applied the CUSUM and CUSUM of squares tests (see Brown, Durbin, and Evans (1975)) that are based on plots against time of the cumulative sums of the regression residuals and residuals squared, respectively. Using the 0.05 significance level, the CUSUM tests suggest model stability while the CUSUM of squares tests suggest instability. Overall, the results based on recursive estimation suggest parameter stability but changing variance over the full sample period. Due to the large number of graphs, they are not shown.

Given the results of our recursive least squares tests, we carried out Wald tests of structural change assuming unequal variances during the three monetary regimes in our sample. Coefficient stability was tested for each bond for each of the five possible regime pairs. The results, reported in Table 3, never reject coefficient stability at the 0.05 level and reject it at the 0.10 level in only one instance. Thus, the Wald tests also suggest parameter stability but changing variance across monetary regimes.

Overall, our specification tests support two conclusions. First, the assumption of coefficient stability over the full sample period is a reasonable one, so our estimates of conditional mean excess returns appear adequate. Second, the volatility of return shocks varies over time, suggesting that an examination of the relation between excess returns and conditional volatility is well motivated. In the next section, we use models of conditional volatility to examine the relation between bond risk and return.

6. The Relation between Excess Returns and Conditional Volatility

In this section, we estimate the empirical relation between bond risk and return. Since the method chosen to model conditional volatility is critical to the results of estimating the risk-return relation in the stock literature, we test three specifications of the conditional variance of bond excess returns. We pay special attention to the decision to include or exclude financial conditioning information in the model of conditional variance, because this decision appears to

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8 We also carry out Wald tests based on the assumption of equal variances in the subperiod. These tests consistently reject model stability.
determine the sign of the estimated risk-return relation for stocks. Our first model estimates conditional variances using predetermined financial conditioning information. Given the strong evidence of ARCH effects in excess returns reported in Table 2, our second model is a simple GARCH-M model. Modeling ARCH effects should improve the efficiency of estimates of the risk-return relation. Our third model incorporates both financial conditioning variables and GARCH effects.

6.1. Instrumental Variables Estimation using Financial Conditioning Information

For each bond maturity, $\tau$, we estimate the following IV regression:

$$R_{t+1} - R_{t} = \alpha_{t,0} + \alpha_{t,1} \varepsilon^2_{t+1} + \mu_{t+1},$$

(11)

Where the $\varepsilon_{t+1}$ are the residuals from the estimation of the equation (10) model of excess returns, the slope coefficient $\alpha_{t,1}$ is the estimate of the relation between the bond’s expected excess return and conditional volatility, and $\mu_{t+1}$ is the error term. For instruments we consider the conditioning variables used to estimate the excess return model and lags of the squared residuals. An initial examination of the relations between the squared residuals and the candidate instruments indicates that the one-month bill rate and six lags of the squared residuals encompass the candidates that are most useful in modeling conditional volatility. We expect shocks to bond excess returns to be correlated across maturities, so we improve the efficiency of our estimates by choosing an estimation method that takes into account the cross-equation correlations in the error terms. We use the Generalized Method of Moments (GMM) to estimate equation (11) simultaneously for all bond maturities. Standard errors are Newey-West heteroskedasticity and autocorrelation consistent.

Results of the system estimation of equation (11) are reported in Table 4. The slope coefficient is significant at the 0.01 level for the 3 month bill and the four bond portfolios of maturities less than or equal to 48 months, significant at the 0.05 level for the 120 month bond,
and statistically insignificant for the 48 to 60 month portfolio and the 240 month bond. Thus, six of our eight maturities produce evidence consistent with a positive relation between bond risk and return. These results contrast with findings in the stock literature of a negative risk-return relation when conditional volatility is modeled using financial conditioning information.

The slope coefficients in Table 4 are estimates of the reward to volatility for each bond maturity. The slope coefficients display a distinct pattern in that they are very high for short-term bonds and low for long-term bonds. The tendency for these estimates of the reward to volatility to decline with bond maturity confirms, with an alternative methodology, the findings of Pilotte and Sterbenz (2006).

To facilitate comparison of the persistence of the conditional variance estimates across differently parameterized models, we regress the conditional variance estimate for each model on a constant and the lagged value of the estimate. These first order autoregressive coefficients are reported for each model that we estimate. For the IV results reported in Table 4, the first order autoregressive coefficient is estimated for the predicted values of the \( \epsilon_{t,t+1}^2 \) from the system estimation of equation (11). These AR(1) coefficients are all close to but slightly less than one, indicating substantial persistence in the conditional variance estimates.

The Q(12)-ARCH statistic reported in Table 4 rejects, at the 0.01 level, the null hypothesis of no ARCH effects in the first 12 lags of the residuals of each equation. The Q(12)-Autocorr and JB statistics are consistent with results reported in Table 2, rejecting the nulls of no autocorrelation and the normality of the residuals. Since GMM requires no distributional assumption, parameter estimates are consistent despite the lack of normally distributed residuals. Because the IV approach to estimating conditional volatility does a poor job of capturing the ARCH effects in our excess return data, there are likely efficiency gains to using GARCH estimation. We use GARCH estimation in the models that follow.
6.2. GARCH-M Estimation

A natural way to estimate the relation between bond risk and return is with the following simple GARCH-M model of conditional variance:

\[ R_{t+1} - R_{t} = \alpha_{t,0} + \alpha_{t,1} \sigma_{t+1}^{2} + \varepsilon_{t+1} \]

\[ \sigma_{t+1}^{2} = \beta_{t,0} + \beta_{t,1} \sigma_{t}^{2} + \beta_{t,2} \varepsilon_{t}^{2} + \eta_{t+1} \]

(12) \hspace{1cm} (13)

Estimation is by the method of maximum likelihood. In light of the evidence in Table 1, that excess returns are not normally distributed due to excess kurtosis, we estimate the GARCH-M system assuming that the conditional distribution for the error term is the Generalized Error Distribution (GED). The GED is less restrictive than the normal as it accommodates kurtosis, although it does not accommodate skewness. The GED distribution nests the Student’s \( t \)-distribution and normal distribution. More detail on the GED appears in the appendix to this paper.

Table 5 contains the results for GARCH-M estimation. For each maturity, the GED parameter differs significantly from 2, the value for the normal distribution, at either the 0.01 or 0.05 significance level. The Lagrange Multiplier ARCH statistics indicate that the model is effective at removing most of the ARCH effects from the regression residuals. The coefficient sum, \( \beta_{t,1} + \beta_{t,2} \), is close to one in every variance equation. A sum of one is indicative of the integrated GARCH (IGARCH) process identified by Engle and Bollerslev (1986), which allows for shocks to have a permanent effect on the conditional variance. An IGARCH process is not covariance-stationary but is strictly stationary under conditions identified in Nelson (1990).

---

9 The GED is a restricted version of the skewed generalized error distribution (SGED). Although it may seem intuitive that a less restrictive distribution is always better, since the non-normality of the error term is not driven by skewness, a loss of efficiency would obtain from over-parameterization of the distribution if specified with the more general SGED.

10 Although not shown, \( \chi^{2} \) distributed goodness-of-fit log-likelihood ratio tests (one degree of freedom) comparing the fits of the GED and the normal distributions for each maturity indicate that the GED provides a statistically-significantly better fit than the normal.

11 Nelson shows that an IGARCH(1,1) process with a positive drift is strictly stationary and ergodic. The unconditional density for such a process is the same for all \( t \).
Similarly, the AR(1) coefficients for the conditional volatility estimates range from 0.93 to 0.97, confirming the presence of substantial persistence in conditional volatility. The persistence in volatility, as measured by the AR(1) coefficients, is similar to that reported in Table 4 for the IV model.

The coefficients on conditional variance in the mean equation are all positive. They are significant at the 0.01 level for all maturities less than or equal to 60 months and significant at the 0.10 level for the 240 month bond. The risk-return relation is insignificant only for the 120 month bond. Unlike the case for the stock risk-return literature, the GARCH-M specification of conditional variance and the IV specification based on financial conditioning information both support the conclusion that there is a positive relation between bond risk and return. The GARCH-M specification is also consistent with the IV specification in that the magnitude of the coefficient on the conditional variance in the mean equation decreases with maturity, providing further evidence that the reward to bond volatility declines with maturity.

6.3. GARCH-M Estimation with Financial Conditioning Information

Our third model of conditional volatility incorporates both financial conditioning variables and GARCH effects:

\[
R_{t,t+1} - R_{f,t} = \alpha_{t,0} + \alpha_{t,1} \sigma^2_{t,t+1} + \gamma_{t,t+1}
\]

\[
\sigma^2_{t,t+1} = \beta_{t,0} + \beta_{t,1} \sigma^2_{t,t-1} + \beta_{t,2} \varepsilon^2_{t,t} + \beta_{t,3} R_{t,t} + \beta_{t,4} (Y_{t,t} - R_{f,t}) + \beta_{t,5} (R_{t,t} - R_{f,t}) + \eta_{t,t+1}
\]

Results, reported in Table 6, indicate that incorporating both financial conditioning variables and GARCH effects in the model of conditional variance provides the strongest evidence of a positive risk-return relation. In the mean equation, the coefficient on the variance term is positive and significant at the 0.01 level for seven of the eight bond maturities and at the 0.05 level for the remaining bond maturity. As is the case for the IV and simple GARCH-M specifications, the magnitude of the coefficient on the conditional variance in the mean equation decreases with
maturity. Thus, all of our models provide evidence that the reward to bond volatility declines with maturity.

An examination of the results for the variance equation indicates that the one-month rate is significant (0.01 level for seven maturities and 0.05 level for one maturity) in explaining the conditional variance of every bond maturity. The significance of the yield spread in explaining conditional variance is mixed. The yield spread is significant at the 0.01 level in three regressions, significant at the 0.10 level in three regressions, and insignificant in two regressions. The lagged excess return is significant (0.05 level) only for the 120 month bond.

In Table 6, the GED parameters differ significantly from the value for the normal distribution (0.01 or 0.05 level) in every regression. The Lagrange Multiplier ARCH statistics indicate that the model is effective at removing most of the ARCH effects from the regression residuals. For each maturity, the inclusion of financial conditioning information in the variance equation significantly increases the value of the log-likelihood function relative to the value reported in Table 5 for simple GARCH-M estimation. The persistence in conditional volatility, as measured by the AR(1) coefficient, is usually close to that reported in Tables 4 and 5 for the IV and simple GARCH models, respectively. The exception is the AR(1) coefficient for the 3 month bill which is much lower in Table 6 than it is in Tables 4 and 5.

6.4. Robustness of Results

As a robustness check, all three models are estimated using the conditional standard deviation rather than the conditional variance to estimate the risk-return relation. While this change does not materially alter our conclusions, there are systematic effects on the p-values for the coefficient on the conditional volatility measure. For IV estimation using financial conditioning information, using the conditional standard deviation tends to raise p-values slightly, providing slightly weaker results. For GARCH-M estimation, both with and without conditioning variables, using the conditional standard deviation tends to lower p-values,
providing slightly stronger results. The preponderance of results remain consistent with a positive risk-return relation.

We also check the robustness of our results to the use of asymmetric GARCH-M models that allow positive and negative returns to have different effects on the conditional volatility. Contrary to the existing evidence for stocks, for which asymmetries are significant determinants of conditional volatility that cause the sign of the risk-return relation to reverse, we find that these asymmetries are insignificant in determining the conditional volatilities of bonds.

We also explore the use of alternatives to the GED distribution for estimating GARCH models when regression residuals are not conditionally normally distributed. We repeat estimation of all GARCH models using the Student’s t-distribution and using the quasi-maximum likelihood method of Bollerslev and Wooldridge (1992). Our conclusions are robust to these changes in the specification of the conditional distribution for errors.

We use GMM system estimation of equation (11) to produce our estimates of the risk-return relation that are based on modeling the conditional variance using only financial conditioning information. Advantages of the GMM estimator are that it takes into account the cross-equation correlations in the error terms and is robust to heteroskedasticity and autocorrelation of unknown form. As a check on the importance of these advantages we also estimate equation (11) using three-stage least squares (3SLS) and single-equation estimation. 3SLS accounts for the cross-equation correlations in the error term and heteroskedasticity, but does not account for autocorrelation in the errors. Single-equation estimation accounts for heteroskedasticity and autocorrelation of unknown form, but not the cross-equation correlations in the error terms. Results for 3SLS are similar, but slightly weaker than GMM estimation. Results for single-equation estimation are substantially weaker, producing positive estimates of the risk-return relation that are significant only for maturities less than or equal to 12 months. Thus, accounting for the cross-equation correlations in the errors produces efficiency gains that
have an important impact on the statistical significance of the estimated relation between bond risk and return.

7. Stability of the Risk-Return Relation

The regression models reported in Tables 4, 5, and 6 assume a time invariant linear relation between the expected excess return and conditional variance. The theoretical model of Section 3 does not restrict the risk-return relation to a stable linear relation. In this section, we evaluate the linearity and stability of the relation between bond risk and return.

7.1. Analysis of Excess Return Model Residuals

As Harvey (2001) notes in his analysis of the risk-return relation for stocks, a simple way to check the linear restriction for any of our models is to examine the relation between the regression error and financial conditioning information. If conditioning information explains variability in expected returns that is not related to conditional volatility, a linear relation between the conditional mean and conditional variance is rejected. Such a finding suggests that the reward to volatility changes over time.

Table 7 reports the results of OLS regressions of residuals from our models on financial conditioning information. For all three models, conditioning variables have explanatory power beyond that of the conditional variance. The explanatory power is greatest for the model where the conditional variance is based only on financial conditioning information. The explanatory power is lower in models where the conditional variance estimates incorporate GARCH effects. At least one conditioning variable is significant in all but one of the residual regressions. Clearly, the conditioning variables capture variation in excess returns that is not related to conditional variance. A strictly linear specification of the relation between the conditional mean and conditional volatility is rejected, which suggests that the reward to volatility changes over time.

Pilote and Sterbenz (2006) find that Sharpe ratios on long-term bonds, but not short-term bonds, vary over the business cycle. Our results differ in indicating that there is time variation in
the reward to volatility for all bond maturities. A potential explanation for the difference in results is that our tests are not tied to the business cycle.

The results for bonds reported in Table 7 are consistent with results that Harvey (2001) reports for stocks. Harvey finds that the rejection of a linear risk-return relation for stocks is robust to changes in the method used to estimate the conditional variance. He also presents graphic evidence that the ratio of conditional mean to conditional volatility for stocks has a distinct business cycle pattern.

7.3. Rolling Correlations between Conditional Means and Conditional Variances

Another approach to examining the relation between risk and return is to directly examine the relation between estimates of the conditional mean and conditional variance. Using a variety of estimators, Harvey (2001) graphs the fitted values of the conditional mean and conditional variance and finds no obvious relation for stocks. Whitelaw (1994) estimates the conditional moments of stock returns using financial conditioning information and finds that the short-term correlation between the conditional mean and volatility varies substantially over time. To facilitate a comparison with existing results for stocks, we follow Whitelaw’s approach. We calculate contemporaneous correlations between estimates of conditional means and conditional variances for each bond maturity over 17-month rolling periods.\(^{12}\)

To get a time series of fitted values, we estimate final models of conditional means and variances for Treasury bond excess returns. Our final model incorporates all aspects of our prior models. The conditional mean is modeled as a function of both the conditional variance and financial conditioning information. The conditional variance incorporates both GARCH effects and financial conditioning information. We first estimate the following GARCH-M model:

\(^{12}\)Whitelaw (1994) chooses the 17-month window to balance the need for reasonably accurate estimates with the need for a period that is short enough to pick up variation over the length of a business cycle.
After the initial estimation, we drop explanatory variables that are not significant at the 0.10 level and re-estimate the model. The final models with only variables that are statistically significant in explaining the conditional mean or conditional variance are reported in Table 8.

An interesting aspect of the results reported in Table 8 is that the GARCH in mean term is not significant for any bond maturity. The effect of the conditional variance on the conditional mean is subsumed by the financial conditioning information. The yield spread and lagged excess return are consistently significant in explaining the excess return. The one-month rate is significant in explaining the excess return only for the 3-month bill. In the variance equation, the GARCH terms and the one-month rate are always significant in explaining the conditional volatility. The yield spread never appears as significant in the variance equation and the lagged excess return is significant only for the 120 month bond. Viewed overall, the results reported in Table 8 indicate that the yield spread and lagged excess return are generally important in predicting conditional means, while the one-month rate and GARCH effects are important in predicting the conditional variances.

Figure 1 presents graphs of the rolling estimates of correlations between the fitted series of conditional excess returns and conditional variances for each bond maturity. The graphs show substantial variation over time in the short-horizon relation between bond risk and return. For longer maturities, both the range of correlations and incidence of negative correlations are similar to those reported by Whitelaw (1994) for stocks. For the shortest maturities, the range of correlations is diminished somewhat, but there remains substantial variation over time and numerous negative correlations.

The graphs in Figure 1 are shaded to show business cycle expansions and contractions. The correlations vary substantially within both expansions and contractions. The graphs show no
obvious business cycle pattern in the relation between bond risk and return; however, they contain only six measured contractions.

For comparison, the graphs of rolling estimates of the correlations for the 240-month bond and 3-month T-bill are superimposed in Figure 2. In that figure, the greater range of correlations for the 240-month bond is obvious. Also obvious is a lower incidence of negative correlations for the 3-month bill, especially since 1990.

To illustrate the co-movement in the risk-return relation across bond maturities, in Table 9 we report correlations between the rolling correlations of each maturity pair. The correlations in Table 9 indicate that time variation in the risk-return relation is similar for adjacent maturities, but differs substantially when the difference in maturity is large. Nevertheless, correlations are positive for every pair of bond maturities.

Overall, our examination of rolling correlations shows substantial instability in the short-horizon relation between bond risk and return. The relation is often negative for each bond maturity. For longer maturities, both the range of correlations and incidence of negative correlations are similar to those reported previously for common stocks. For shorter maturities the range is diminished somewhat; however, the rolling correlations for all bond maturities do tend to move together.

8. Conclusions

We find a significant positive relation between bond excess returns and conditional volatility. This finding is not very sensitive to the method used to estimate conditional volatility. Our results contrast with prior studies of the stock risk-return relation, where the specification of the conditional volatility influences inferences regarding the relation between excess returns and conditional volatility. In the stock literature, the use of financial conditioning information to model conditional volatility produces negative estimates of the risk-return relation. To the contrary, we find that the use of financial conditioning information alone produces positive
estimates of the risk-return relation for bonds. When added to the volatility equation of GARCH-M models, financial conditioning information reinforces rather than reverses the positive estimate of the risk-return relation. Thus, our strongest evidence of a positive relation between bond risk and return is produced by a model that incorporates both financial conditioning information and GARCH effects in estimating the conditional volatility. Where the stock literature can be characterized as a search for a positive relation between risk and return, a positive risk-return relation is conspicuous for bonds.

Our results are good news for investors who chose to invest predominantly in bonds, as they indicate that, over our sample period, bond excess returns adjusted period-by-period to changes in the risks of holding bonds. From an asset allocation perspective, our results may be viewed as a disappointment. A positive, rather than negative, risk-return relation indicates that bonds are not an intertemporal hedging asset as suggested by Merton (1980). Our results also confirm prior findings that the reward to volatility is much higher for short-term than for long-term bonds, so that superior combinations of risk and return can be achieved by highly leveraged investments in short-maturity bonds.

Some of our results are similar to prior results for stocks. These involve our examination of the linearity and stability of the risk-return relation. Results of this examination suggest that the short-horizon relation between bond risk and return varies substantially over time. Thus, the reward to risk for bonds, like that of stocks, appears to be time varying. This evidence of misspecification in the time invariant linear model of bond risk and return suggests that richer models of bond risk and return, such as regime-switching models, are well motivated.
Appendix

The Generalized Error Distribution and Nested Distributions

The GED and two of its nested distributions, the Student’s t and normal, are:

\[
GED(\varepsilon, \beta, \sigma, k) = \frac{C}{\sigma} \exp\left[-\frac{1}{\theta^k \sigma^k} |\varepsilon|^k\right]
\]

\[
t(\varepsilon, \beta, \sigma, n) = \frac{C}{\sigma} \left[1 + \frac{1}{((n + 1)/2) \theta^2 \sigma^2 |\varepsilon|^2}\right]^{\left(\frac{n+1}{2}\right)}
\]

\[
Normal(\varepsilon, \beta, \sigma) = \frac{C}{\sigma} \exp\left[-\frac{1}{\theta^2 \sigma^2} |\varepsilon|^2\right]
\]

where \( \varepsilon \) is the error term, \( \beta \) is a vector of regression parameters, \( \sigma \) is the standard deviation of the error term, and \( k \) and \( n \) are kurtosis parameters. \( C \) and \( \theta \) are normalizing constants. The GED distribution is less restrictive than the normal as it accommodates fat tails, or, kurtosis, although it does not accommodate skewness. The GED allows the MLE to estimate the optimal \( k \), where \( k \) is fixed at 2 for the normal distribution. Student’s t-distribution also accommodates kurtosis with a flexible parameter, \( n \). An inspection of Table 1 shows that kurtosis, not skewness, is the determining factor for the non-normality of bond excess returns. A goodness of fit test using the chi-square distributed likelihood ratio test can be performed among nesting distributions, with the degrees of freedom equal to the difference in the number of parameters (one degree of freedom for the GED and normal goodness of fit test). The test statistic is \( 2(l_{\text{GED}} - l_{\text{Normal}}) \) where \( l \) is the value of the log-likelihood function.
References


Table 1: Descriptive Statistics for Treasury Bond Excess Returns

The time series is from January 1961 to December 2002 with 504 observations. The Jarque-Bera (JB) statistic is a goodness-of-fit measure of the departure of the distribution of a data series from normality, based on the levels of skewness and excess kurtosis. The JB statistic is \( \chi^2 \) distributed with 2 degrees of freedom. The Q(12) statistic tests for autocorrelation in the first 12 lags. It is \( \chi^2 \) distributed with 12 degrees of freedom based on the number of lags tested. The autocorrelation coefficient is denoted by \( \rho_t \), where \( t \) is the lag, in months.

<table>
<thead>
<tr>
<th>Maturity (months)</th>
<th>Panel A: Monthly Excess Return ( (R_{t+1} - R_f) )</th>
<th>Panel B: Squared Excess Returns ( (R_{t+1} - R_f)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean ((\times 100))</td>
<td>Std. Dev. ((\times 100))</td>
<td>Skewness</td>
</tr>
<tr>
<td>( \tau \approx 3 )</td>
<td>0.0581</td>
<td>0.0966</td>
</tr>
<tr>
<td>0&lt;( \tau \leq 12 )</td>
<td>0.0692</td>
<td>0.2746</td>
</tr>
<tr>
<td>12&lt;( \tau \leq 24 )</td>
<td>0.1093</td>
<td>0.6825</td>
</tr>
<tr>
<td>24&lt;( \tau \leq 36 )</td>
<td>0.1324</td>
<td>1.0291</td>
</tr>
<tr>
<td>36&lt;( \tau \leq 48 )</td>
<td>0.1452</td>
<td>1.2727</td>
</tr>
<tr>
<td>48&lt;( \tau \leq 60 )</td>
<td>0.1355</td>
<td>1.4758</td>
</tr>
<tr>
<td>( \tau \approx 120 )</td>
<td>0.1493</td>
<td>2.2206</td>
</tr>
<tr>
<td>( \tau \approx 240 )</td>
<td>0.1606</td>
<td>2.8060</td>
</tr>
</tbody>
</table>

***JB or Q(12) test significant at 0.01 level, one-tailed test.
** JB or Q(12) test significant at 0.05 level, one-tailed test.
* JB or Q(12) test significant at 0.10 level, one-tailed test.
Table 2: Ordinary Least Squares (OLS) Regressions of Monthly Excess Returns on Conditioning Variables

The time series is from January 1961 to December 2002. Regressions of the monthly excess return ($R_{t+1} - R_f$) on the beginning of period risk-free rate ($R_f$), the beginning-of-period yield spread ($Y_{t,-} - R_f$), and, the one-month lag of the excess return ($R_t - R_{f,t-1}$). The Jarque-Bera (JB) statistic is a goodness-of-fit measure of the departure of the distribution of the regression residuals from normality. The JB statistic is distributed with 2 degrees of freedom. The White statistic is a test for heteroskedasticity that is distributed with 6 degrees of freedom. The Breusch-Godfrey Lagrange Multiplier (LM-Serial-Corr.) statistic is a test for serial correlation that is distributed with 12 degrees of freedom due to the test for serial correlation for up to 12 lags. Engle’s Lagrange Multiplier ARCH statistic (LM-ARCH) is a test for ARCH effects in the residuals. It is distributed with 12 degrees of freedom due to the test for ARCH effects for 12 lags. Newey-West autocorrelation and heteroskedasticity consistent standard errors are in parentheses.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Constant</th>
<th>$R_{f,t}$</th>
<th>$Y_{t,-} - R_{f,t}$</th>
<th>$R_t - R_{f,t-1}$</th>
<th>$R^2$</th>
<th>JB</th>
<th>White-Hetero.</th>
<th>LM-Serial Corr.</th>
<th>LM-ARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau \approx 3$</td>
<td>-0.000</td>
<td>0.073**</td>
<td>0.854***</td>
<td>0.207***</td>
<td>0.25</td>
<td>3,304.2***</td>
<td>145.1***</td>
<td>44.7***</td>
<td>105.8***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.031)</td>
<td>(0.179)</td>
<td>(0.067)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0 &lt; \tau \leq 12$</td>
<td>-0.000</td>
<td>0.042</td>
<td>0.993***</td>
<td>0.176***</td>
<td>0.09</td>
<td>5,492.7***</td>
<td>119.5***</td>
<td>67.3***</td>
<td>88.1***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.091)</td>
<td>(0.258)</td>
<td>(0.061)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$12 &lt; \tau \leq 24$</td>
<td>-0.001</td>
<td>0.162</td>
<td>1.423***</td>
<td>0.181***</td>
<td>0.07</td>
<td>4,063.2***</td>
<td>95.8***</td>
<td>43.5***</td>
<td>77.3***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.229)</td>
<td>(0.529)</td>
<td>(0.050)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$24 &lt; \tau \leq 36$</td>
<td>-0.002</td>
<td>0.245</td>
<td>1.791**</td>
<td>0.137***</td>
<td>0.05</td>
<td>2,943.0***</td>
<td>90.8***</td>
<td>30.9***</td>
<td>73.6***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.321)</td>
<td>(0.721)</td>
<td>(0.048)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$36 &lt; \tau \leq 48$</td>
<td>-0.003*</td>
<td>0.282</td>
<td>2.253***</td>
<td>0.130**</td>
<td>0.05</td>
<td>608.7***</td>
<td>98.6***</td>
<td>18.7*</td>
<td>88.3***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.364)</td>
<td>(0.800)</td>
<td>(0.051)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$48 &lt; \tau \leq 60$</td>
<td>-0.004*</td>
<td>0.339</td>
<td>2.300***</td>
<td>0.129***</td>
<td>0.05</td>
<td>425.8***</td>
<td>92.6***</td>
<td>16.4</td>
<td>83.9***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.410)</td>
<td>(0.869)</td>
<td>(0.046)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau \approx 120$</td>
<td>-0.007**</td>
<td>0.799</td>
<td>3.551***</td>
<td>0.080*</td>
<td>0.03</td>
<td>28.0***</td>
<td>73.6***</td>
<td>12.4</td>
<td>83.4***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.572)</td>
<td>(1.293)</td>
<td>(0.047)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau \approx 240$</td>
<td>-0.007**</td>
<td>0.775</td>
<td>4.064***</td>
<td>0.048</td>
<td>0.03</td>
<td>128.2***</td>
<td>70.2***</td>
<td>22.5*</td>
<td>61.1***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.687)</td>
<td>(1.227)</td>
<td>(0.051)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*** Significant at 0.01 level, two-tailed test for regression parameters, one-tailed test for JB, White, and LM tests.
** Significant at 0.05 level, two-tailed test for regression parameters, one-tailed test for JB, White, and LM tests.
* Significant at 0.10 level, two-tailed test for regression parameters, one-tailed test for JB, White, and LM tests.
### Table 3: Tests of Coefficient Stability across Monetary Regimes

Results of Wald tests based on OLS regressions of the monthly excess return ($R_{\tau,t+1} - R_{f,t}$) on the beginning of period risk-free rate ($R_{f,t}$), the beginning-of-period yield spread ($Y_{\tau,t} - R_{f,t}$), and, the one-month lag of the excess return ($R_{\tau,t} - R_{f,t-1}$). We test for regression coefficient stability when sub-sample period variances are unequal. The Wald statistic is $\chi^2$ distributed with 4 degrees of freedom. The null hypothesis tested is that the regression coefficients are equal among regimes. The values below the Wald statistics in parentheses are the p-values (probability levels of significance) for the associated Wald statistic.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>$1/61 - 10/79$ versus $11/79 - 10/82$</th>
<th>$1/61 - 10/79$ versus $11/82 - 12/02$</th>
<th>$1/61 - 10/79$ versus $11/79 - 12/02$</th>
<th>$1/61 - 10/82$ versus $11/82 - 12/02$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau = 3$</td>
<td>6.521 (0.16)</td>
<td>5.495 (0.24)</td>
<td>3.772 (0.44)</td>
<td>8.981* (0.06)</td>
</tr>
<tr>
<td>$0 &lt; \tau \leq 12$</td>
<td>6.546 (0.16)</td>
<td>6.889 (0.14)</td>
<td>7.395 (0.12)</td>
<td>3.211 (0.52)</td>
</tr>
<tr>
<td>$12 &lt; \tau \leq 24$</td>
<td>5.723 (0.22)</td>
<td>5.571 (0.23)</td>
<td>5.950 (0.20)</td>
<td>3.760 (0.44)</td>
</tr>
<tr>
<td>$24 &lt; \tau \leq 36$</td>
<td>3.895 (0.42)</td>
<td>3.589 (0.46)</td>
<td>6.789 (0.15)</td>
<td>4.674 (0.32)</td>
</tr>
<tr>
<td>$36 &lt; \tau \leq 48$</td>
<td>4.059 (0.40)</td>
<td>2.982 (0.56)</td>
<td>6.566 (0.16)</td>
<td>6.100 (0.19)</td>
</tr>
<tr>
<td>$48 &lt; \tau \leq 60$</td>
<td>4.615 (0.33)</td>
<td>3.479 (0.48)</td>
<td>5.233 (0.26)</td>
<td>5.321 (0.26)</td>
</tr>
<tr>
<td>$\tau = 120$</td>
<td>1.868 (0.76)</td>
<td>1.181 (0.88)</td>
<td>1.409 (0.84)</td>
<td>2.081 (0.72)</td>
</tr>
<tr>
<td>$\tau = 240$</td>
<td>5.290 (0.26)</td>
<td>2.943 (0.57)</td>
<td>5.859 (0.21)</td>
<td>7.314 (0.12)</td>
</tr>
</tbody>
</table>

Note: Given two period vectors of regression coefficients, $\beta_1$ and $\beta_2$, and associated covariance matrices, $V_1$ and $V_2$, the Wald statistic test is $(\beta_1 - \beta_2)' (V_1 + V_2)^{-1} (\beta_1 - \beta_2)$. If the vectors $\beta_1$ and $\beta_2$ are independently normally distributed, then $(\beta_1 - \beta_2)$ has a zero mean and variance equal to $(V_1 + V_2)$.
Table 4: Instrumental Variables Estimation of Risk-Return Relation for Treasury Bonds

Generalized method of moments (GMM) system estimation incorporates the use of instrumental variables and considers the cross-equation correlations in the error terms. The following system of equations is estimated:

\[ R_{t+1} - R_{f,t} = \alpha_{t,0} + \alpha_{t,1} \varepsilon_{t+1} + \mu_{t+1} \]

where, \( \tau \) is the number of months of bond maturity: \( \tau = 3, \ 0 < \tau \leq 12, \ 0 < \tau \leq 24, \ 0 < \tau \leq 36, \ 0 < \tau \leq 48, \ 0 < \tau \leq 60, \ \approx 120, \ \approx 240, \) time \( t = 1,504 \) represents the beginning of months from January 1961 to December 2002, \( \varepsilon_{t+1} \) is the residual from the OLS regressions in Table 2, and \( \mu_{t+1} \) is the error term. The instrumental variables are the one-month return on the one month T-Bill \( (R_{f,t}) \) and the first six monthly lags of the squared residuals. The Q(12)-ARCH statistic tests the null hypothesis of no ARCH effects in the first 12 lags of the system estimated residuals of each equation. The Q(12)-Autocorr statistic tests the null hypothesis that there is no serial correlation in the first 12 lags of residuals. The Jarque-Bera (JB) statistic is a goodness-of-fit measure of the departure of the distribution of the regression residuals from normality. The JB statistic is \( \chi^2 \) distributed with 2 degrees of freedom. The AR(1) coefficient is the first order autoregressive coefficient for the fitted values of \( \varepsilon_{t+1}^2 \). Newey-West heteroskedasticity and autocorrelation consistent standard errors are in parentheses.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Constant ((\times 10^4))</th>
<th>Slope</th>
<th>Q(12)-ARCH</th>
<th>Q(12)-Autocorr.</th>
<th>JB</th>
<th>AR(1) for Predicted ( \varepsilon_{t+1}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau=3 )</td>
<td>4.210*** ((0.298))</td>
<td>253.097*** ((14.294))</td>
<td>138.0***</td>
<td>79.1***</td>
<td>5,254.5*** ((0.952))</td>
<td>0.952*** ((0.014))</td>
</tr>
<tr>
<td>( 0&lt;\tau\leq12 )</td>
<td>5.700*** ((0.679))</td>
<td>17.399*** ((2.969))</td>
<td>450.2***</td>
<td>63.6***</td>
<td>1,141.6*** ((0.952))</td>
<td>0.952*** ((0.014))</td>
</tr>
<tr>
<td>( 12&lt;\tau\leq24 )</td>
<td>8.300*** ((1.750))</td>
<td>6.191*** ((1.139))</td>
<td>232.8***</td>
<td>48.9***</td>
<td>1,733.6*** ((0.953))</td>
<td>0.953*** ((0.014))</td>
</tr>
<tr>
<td>( 24&lt;\tau\leq36 )</td>
<td>9.850*** ((2.650))</td>
<td>3.616*** ((0.779))</td>
<td>195.9***</td>
<td>34.3***</td>
<td>1,281.2*** ((0.951))</td>
<td>0.951*** ((0.014))</td>
</tr>
<tr>
<td>( 36&lt;\tau\leq48 )</td>
<td>11.410*** ((3.440))</td>
<td>2.177*** ((0.784))</td>
<td>209.2***</td>
<td>25.8***</td>
<td>470.0*** ((0.966))</td>
<td>0.966*** ((0.012))</td>
</tr>
<tr>
<td>( 48&lt;\tau\leq60 )</td>
<td>13.520*** ((4.260))</td>
<td>-0.394 ((0.829))</td>
<td>170.9***</td>
<td>26.7***</td>
<td>370.8*** ((0.969))</td>
<td>0.969*** ((0.013))</td>
</tr>
<tr>
<td>( \tau=120 )</td>
<td>6.740 ((7.470))</td>
<td>1.836*** ((0.849))</td>
<td>187.5***</td>
<td>15.3</td>
<td>25.7*** ((0.969))</td>
<td>0.969*** ((0.011))</td>
</tr>
<tr>
<td>( \tau=240 )</td>
<td>15.530* ((8.820))</td>
<td>-0.085 ((0.593))</td>
<td>132.7***</td>
<td>26.0**</td>
<td>102.2*** ((0.982))</td>
<td>0.982*** ((0.009))</td>
</tr>
</tbody>
</table>

***Significant at 0.01 level, two-tailed test for regression parameters, one-tail test for Q and JB statistics.
** Significant at 0.05 level, two-tailed test for regression parameters, one-tail test for Q and JB statistics.
* Significant at 0.10 level, two-tailed test for regression parameters, one-tail test for Q and JB statistics.
### Table 5: GARCH-M Estimation of Risk-Return Relation for Treasury Bonds

The results below are the GARCH-M regressions for the monthly excess return on the T-Bond ($R_{t+1} - R_f$) with conditional variance in the mean equation. The estimated models are:

\[
R_{t+1} - R_f = \alpha_{t,0} + \alpha_{t,1} \sigma_{t+1}^2 + \epsilon_{t+1}
\]

\[
\sigma_{t+1}^2 = \beta_{t,0} + \beta_{t,1} \sigma_{t,1}^2 + \beta_{t,2} \epsilon_{t,j}^2 + \eta_{t+1}
\]

The time series is from January 1961 to December 2002 with 504 observations. The conditional distribution for the error term is the generalized error distribution (GED) to address non-normality of the errors, where the GED parameter ($k$) is the kurtosis parameter that accommodates fat tails. The GED nests the normal distribution and becomes the normal if $k$ is equal to 2. Engle’s Lagrange Multiplier ARCH statistic (LM-ARCH) is a test for ARCH effects in the residuals. It is $\chi^2$ distributed with 12 degrees of freedom due to the test for ARCH effects for 12 lags. Log-L is the value of the log likelihood function. The AR(1) coefficient is the first order autoregressive coefficient for the fitted values of $\sigma_{t+1}^2$. Standard errors are in parentheses.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Mean Equation</th>
<th>Variance Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant $(\times 10)$</td>
<td>$\sigma_{t+1}^2$</td>
</tr>
<tr>
<td>$\tau \approx 3$</td>
<td>0.003***</td>
<td>184.579***</td>
</tr>
<tr>
<td>$0 &lt; \tau \leq 12$</td>
<td>0.004***</td>
<td>38.496***</td>
</tr>
<tr>
<td>$12 &lt; \tau \leq 24$</td>
<td>0.003</td>
<td>15.056**</td>
</tr>
<tr>
<td>$24 &lt; \tau \leq 36$</td>
<td>0.002</td>
<td>8.756**</td>
</tr>
<tr>
<td>$36 &lt; \tau \leq 48$</td>
<td>0.001</td>
<td>7.607**</td>
</tr>
<tr>
<td>$48 &lt; \tau \leq 60$</td>
<td>-0.005</td>
<td>6.809**</td>
</tr>
<tr>
<td>$\tau \approx 120$</td>
<td>-0.000</td>
<td>2.559</td>
</tr>
<tr>
<td>$\tau \approx 240$</td>
<td>-0.009</td>
<td>3.171*</td>
</tr>
</tbody>
</table>

***Significant at 0.01 level, two-tailed test for regression and GED parameters, one-tailed test for LM-ARCH.
** Significant at 0.05 level, two-tailed test for regression and GED parameters, one-tailed test for LM-ARCH.
* Significant at 0.10 level, two-tailed test for regression and GED parameters, one-tailed test for LM-ARCH.
The following GARCH–M models are estimated:

\[ R_{t+1} - R_{f,t} = \alpha_{t,0} + \alpha_{t,1} \sigma^2_{t+1} + \gamma_{t,1} \]

\[ \sigma^2_{t+1} = \beta_{t,0} + \beta_{t,2} R^2_{t} + \beta_{t,4} (Y_{t} - R_{f,t}) + \beta_{t,6} (R_{t+1} - R_{f,t+1}) + \nu_{t+1} \]

The time series is from January 1961 to December 2002 with 504 observations. These regression models estimate the relation between the excess return \((R_{t+1} - R_{f,t})\) and its conditional variance, where the conditioning variables include the beginning of period monthly return on the 1 month T-Bill \((R_{f,t})\), the beginning of period yield spread \((Y_{t} - R_{f,t})\), and the one-month lag of excess return \((R_{t+1} - R_{f,t+1})\). The conditional distribution for the error term is the generalized error distribution (GED) to address non-normality of the errors, where the GED parameter \((k)\) is the kurtosis parameter that accommodates fat tails. The GED nests the normal distribution and becomes the normal if \(k = 2\). Engle’s Lagrange Multiplier ARCH statistic (LM-ARCH) is a test for ARCH effects in the residuals. It is \(\chi^2\) distributed with 12 degrees of freedom due to the test for ARCH effects for 12 lags. Log-L is the value of the log likelihood function. \(AR(1)\) is the first order autoregressive coefficient for the fitted values of \(\sigma^2_{t+1}\). Standard errors are in parentheses.

### Table 6: GARCH-M Estimation of Risk-Return Relation with Conditioning Variables in the Variance Equation

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Constant ((\times 10^4))</th>
<th>(\sigma^2_{t+1}) ((\times 10^4))</th>
<th>Constant ((\times 10^6))</th>
<th>(\sigma^2_{t} ) ((\times 10^4))</th>
<th>(\epsilon^2_{t} ) ((\times 10^4))</th>
<th>(R_{f,t} ) ((\times 10^6))</th>
<th>(Y_{t} - R_{f,t} ) ((\times 10^4))</th>
<th>(R_{t+1} - R_{f,t+1} ) ((\times 10^6))</th>
<th>GED Parameter</th>
<th>LM-ARCH</th>
<th>Log-L</th>
<th>AR(1) coefficient for (\sigma^2_{t+1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau \approx 3)</td>
<td>2.050*** (0.242)</td>
<td>279.203*** (69.682)</td>
<td>-0.034 (0.025)</td>
<td>0.127 (0.080)</td>
<td>0.499*** (0.134)</td>
<td>3.140*** (1.120)</td>
<td>6.350*** (1.910)</td>
<td>1.330 (1.060)</td>
<td>0.911*** (0.085)</td>
<td>8.7</td>
<td>3,029.7</td>
<td>0.483*** (0.039)</td>
</tr>
<tr>
<td>(0 &lt; \tau \leq 12)</td>
<td>3.280*** (0.08)</td>
<td>59.342*** (16.995)</td>
<td>-0.161* (0.084)</td>
<td>0.832*** (0.043)</td>
<td>0.121*** (0.034)</td>
<td>0.671** (0.031)</td>
<td>0.109 (0.078)</td>
<td>-0.304 (0.514)</td>
<td>1.510*** (0.146)</td>
<td>26.8***</td>
<td>2,473.3</td>
<td>0.972*** (0.011)</td>
</tr>
<tr>
<td>(12 &lt; \tau \leq 24)</td>
<td>0.118 (0.218)</td>
<td>23.255*** (7.463)</td>
<td>-1.820* (0.933)</td>
<td>0.830*** (0.053)</td>
<td>0.114*** (0.034)</td>
<td>6.940** (3.290)</td>
<td>7.960 (5.810)</td>
<td>0.150 (1.810)</td>
<td>1.545*** (0.141)</td>
<td>24.0**</td>
<td>1,952.2</td>
<td>0.966*** (0.012)</td>
</tr>
<tr>
<td>(24 &lt; \tau \leq 36)</td>
<td>1.710 (3.290)</td>
<td>13.065** (5.398)</td>
<td>-6.810*** (1.550)</td>
<td>0.762*** (0.057)</td>
<td>0.132*** (0.038)</td>
<td>25.960*** (7.560)</td>
<td>21.040*** (4.000)</td>
<td>2.590 (3.620)</td>
<td>1.573** (0.138)</td>
<td>16.0</td>
<td>1,722.1</td>
<td>0.935*** (0.016)</td>
</tr>
<tr>
<td>(36 &lt; \tau \leq 48)</td>
<td>-3.900 (2.620)</td>
<td>13.841*** (3.683)</td>
<td>-16.300*** (8.010)</td>
<td>0.782*** (0.082)</td>
<td>0.126*** (0.044)</td>
<td>49.090** (23.570)</td>
<td>69.180* (40.330)</td>
<td>-0.331 (0.392)</td>
<td>1.583** (0.151)</td>
<td>13.5</td>
<td>1,598.9</td>
<td>0.957*** (0.013)</td>
</tr>
<tr>
<td>(48 &lt; \tau \leq 60)</td>
<td>-11.480*** (2.670)</td>
<td>12.136*** (3.074)</td>
<td>-28.300*** (13.100)</td>
<td>0.796*** (0.078)</td>
<td>0.109*** (0.039)</td>
<td>79.870** (36.120)</td>
<td>113.170* (62.850)</td>
<td>-5.540 (3.900)</td>
<td>1.500*** (0.140)</td>
<td>17.6</td>
<td>1,513.0</td>
<td>0.967*** (0.011)</td>
</tr>
<tr>
<td>(\tau = 120)</td>
<td>-9.190* (5.160)</td>
<td>8.281*** (2.206)</td>
<td>-38.100*** (1.900)</td>
<td>0.827*** (0.027)</td>
<td>0.125*** (0.030)</td>
<td>112.720*** (2.820)</td>
<td>116.750*** (23.350)</td>
<td>-8.860** (4.220)</td>
<td>1.768** (0.171)</td>
<td>6.0</td>
<td>1,293.8</td>
<td>0.966*** (0.011)</td>
</tr>
<tr>
<td>(\tau = 240)</td>
<td>-14.620 (9.80)</td>
<td>3.799*** (1.881)</td>
<td>-46.800*** (20.500)</td>
<td>0.881*** (0.039)</td>
<td>0.079*** (0.028)</td>
<td>147.890*** (64.600)</td>
<td>134.020* (76.850)</td>
<td>-10.710 (6.880)</td>
<td>1.695** (0.168)</td>
<td>13.0</td>
<td>1,165.7</td>
<td>0.983*** (0.008)</td>
</tr>
</tbody>
</table>

***Significant at 0.01 level. **Significant at 0.05 level. *Significant at 0.10 level. The regression and GED parameters are two-tailed tests. The LM-ARCH is a one-tail test.
Table 7: Analysis of Residuals from Models of the Bond Risk-Return Relation

Residuals are from the excess return regressions reported in Tables 4, 5, and 6, where the conditional volatility is modeled using financial conditioning information in Table 4, a simple GARCH-M model in Table 5, and a GARCH-M model with financial conditioning information included in the conditional variance equation in Table 7. The residuals from each model of the risk-return relation are regressed on the beginning of period risk-free rate ($R_{f,t}$), the beginning-of-period yield spread ($Y_{\tau,t} - R_{f,t}$), and, the one-month lag of the excess return ($R_{\tau,t} - R_{f,t-1}$). Results are for OLS estimation with Newey-West autocorrelation and heteroskedasticity consistent standard errors reported in parentheses.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Residuals from risk-return model with conditional volatility estimates based on financial conditioning information</th>
<th>Residuals from risk-return model with conditional volatility estimates based on simple GARCH-M model</th>
<th>Residuals from risk-return model with conditional volatility estimates based on GARCH-M with financial conditioning information in the variance equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Con. ($\times 10^3$)</td>
<td>$R_{\tau,t}$</td>
<td>$Y_{\tau,t} - R_{f,t}$</td>
</tr>
<tr>
<td>τ=3</td>
<td>-1.430 (0.914)</td>
<td>-0.060*** (0.018)</td>
<td>0.927*** (0.102)</td>
</tr>
<tr>
<td>0&lt;τ≤12</td>
<td>-7.750*** (2.950)</td>
<td>-0.053 (0.056)</td>
<td>1.031*** (0.186)</td>
</tr>
<tr>
<td>12&lt;τ≤24</td>
<td>-16.840** (7.950)</td>
<td>-0.023 (0.140)</td>
<td>1.492*** (0.369)</td>
</tr>
<tr>
<td>24&lt;τ≤36</td>
<td>-25.730** (12.780)</td>
<td>0.0230 (0.212)</td>
<td>1.870*** (0.494)</td>
</tr>
<tr>
<td>36&lt;τ≤48</td>
<td>-37.660** (16.390)</td>
<td>0.1170 (0.266)</td>
<td>2.279*** (0.567)</td>
</tr>
<tr>
<td>48&lt;τ≤60</td>
<td>-49.210** (19.380)</td>
<td>0.3600 (0.311)</td>
<td>2.284*** (0.626)</td>
</tr>
<tr>
<td>τ=120</td>
<td>-67.280** (31.860)</td>
<td>0.5170 (0.502)</td>
<td>3.372*** (0.957)</td>
</tr>
<tr>
<td>τ=240</td>
<td>-89.450** (39.430)</td>
<td>0.7860 (0.634)</td>
<td>4.056*** (1.066)</td>
</tr>
</tbody>
</table>

***Significant at 0.01 level, two-tailed test.
**Significant at 0.05 level, two-tailed test.
*Significant at 0.10 level, two-tailed test.
Table 8: Final Models of Conditional Means and Conditional Variances for Treasury Bond Returns

The initial estimated models are:

\[
R_{t+1} - R_{t} = \alpha_{t} + \alpha_{t} \sigma_{t+1}^{2} + \alpha_{t} R_{t} + \alpha_{t} (Y_{t} - R_{t}) + \alpha_{t} (R_{t} - R_{t+1}) + \epsilon_{t+1},
\]

\[
\sigma_{t+1}^{2} = \beta_{t} + \beta_{t} \sigma_{t}^{2} + \beta_{t} \sigma_{t+1}^{2} + \beta_{t} R_{t} + \beta_{t} (Y_{t} - R_{t}) + \beta_{t} (R_{t} - R_{t+1}) + \tau_{t+1},
\]

The time series is from January 1961 to December 2002 with 504 observations. The insignificant variables were dropped to obtain the final estimated models reported below. The initial regression models include the conditional variance in the mean equation, and both the mean and conditional variance equations initially include the beginning of period monthly return on the 1 month T-Bill \((R_{t})\), the beginning of period yield spread \((Y_{t} - R_{t})\), and the one-month lag of excess return \((R_{t} - R_{t-1})\) as conditioning variables. The conditional distribution for the error term for the estimations is the generalized error distribution (GED) to address non-normality of the errors. The GED parameter \(k\) is the kurtosis parameter that accommodates fat tails and becomes the normal distribution if \(k\) is equal to 2. Engle’s Lagrange Multiplier ARCH statistic (LM-ARCH) is a test for ARCH effects in the residuals. It is distributed with 12 degrees of freedom due to the test for ARCH effects for 12 lags. Log-L is the value of the log likelihood function. AR(1) is the first order autoregressive coefficient for the fitted values of \(\sigma_{t+1}^{2}\). Standard errors are in parentheses.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Constant ((\times 10^4))</th>
<th>(Y_{t} - R_{t}) ((\times 10^6))</th>
<th>(R_{t} - R_{t-1}) ((\times 10^4))</th>
<th>(R_{t}) ((\times 10^4))</th>
<th>(R_{t} - R_{t-1}) ((\times 10^4))</th>
<th>(\sigma_{t+1}^{2}) ((\times 10^4))</th>
<th>(\epsilon_{t+1}) ((\times 10^4))</th>
<th>(R_{t}) ((\times 10^4))</th>
<th>(R_{t} - R_{t-1}) ((\times 10^4))</th>
<th>GED Parameter</th>
<th>LM-ARCH</th>
<th>Log-L</th>
<th>AR(1) coefficient for (\sigma_{t+1}^{2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau=3)</td>
<td>0.057 ((0.363))</td>
<td>0.956*** ((0.058))</td>
<td>0.995*** ((0.035))</td>
<td>0.020*** ((0.010))</td>
<td>0.012*** ((0.042))</td>
<td>0.709*** ((0.058))</td>
<td>0.246*** ((0.072))</td>
<td>0.087*** ((0.024))</td>
<td>1.085*** ((0.090))</td>
<td>7.0</td>
<td>3,110.1</td>
<td>0.899*** ((0.020))</td>
<td></td>
</tr>
<tr>
<td>(0&lt;\tau\leq12)</td>
<td>-1.630* ((0.984))</td>
<td>0.848*** ((0.121))</td>
<td>0.164*** ((0.045))</td>
<td>0.015** ((0.072))</td>
<td>0.812*** ((0.041))</td>
<td>0.155*** ((0.044))</td>
<td>0.747*** ((0.289))</td>
<td>1.381*** ((0.130))</td>
<td>24.6**</td>
<td>2,496.1</td>
<td>0.942** ((0.015))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(12&lt;\tau\leq24)</td>
<td>-6.150** ((2.700))</td>
<td>1.352*** ((0.276))</td>
<td>0.176*** ((0.045))</td>
<td>-1.510** ((0.425))</td>
<td>0.849*** ((0.033))</td>
<td>0.116*** ((0.032))</td>
<td>6.510*** ((1.770))</td>
<td>1.436*** ((0.132))</td>
<td>28.9***</td>
<td>1,964.4</td>
<td>0.956*** ((0.013))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(24&lt;\tau\leq36)</td>
<td>-10.010** ((4.150))</td>
<td>1.783*** ((0.380))</td>
<td>0.126*** ((0.044))</td>
<td>-4.250*** ((0.791))</td>
<td>0.860*** ((0.027))</td>
<td>0.107*** ((0.028))</td>
<td>17.290*** ((3.550))</td>
<td>1.416*** ((0.134))</td>
<td>16.0</td>
<td>1,733.8</td>
<td>0.961** ((0.012))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(36&lt;\tau\leq48)</td>
<td>-15.110*** ((5.250))</td>
<td>2.077*** ((0.453))</td>
<td>0.117*** ((0.044))</td>
<td>-8.050*** ((1.330))</td>
<td>0.848*** ((0.027))</td>
<td>0.113*** ((0.030))</td>
<td>33.080*** ((6.000))</td>
<td>1.515*** ((0.156))</td>
<td>13.6</td>
<td>1,604.3</td>
<td>0.969*** ((0.011))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(48&lt;\tau\leq60)</td>
<td>-19.240*** ((5.430))</td>
<td>2.212*** ((0.483))</td>
<td>0.128*** ((0.041))</td>
<td>-8.750*** ((1.340))</td>
<td>0.898*** ((0.022))</td>
<td>0.077*** ((0.024))</td>
<td>33.660*** ((5.410))</td>
<td>1.434*** ((0.138))</td>
<td>18.7*</td>
<td>1,519.0</td>
<td>0.983*** ((0.009))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\tau=120)</td>
<td>-24.530*** ((7.360))</td>
<td>2.864*** ((0.701))</td>
<td>0.107*** ((0.040))</td>
<td>-13.300*** ((0.368))</td>
<td>0.880*** ((0.024))</td>
<td>0.116*** ((0.030))</td>
<td>48.520*** ((2.30))</td>
<td>1.688*** ((0.168))</td>
<td>15.5</td>
<td>1,302.1</td>
<td>0.978*** ((0.009))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\tau=240)</td>
<td>-31.730*** ((11.620))</td>
<td>3.487*** ((0.802))</td>
<td>0.076*** ((0.040))</td>
<td>-20.300*** ((5.740))</td>
<td>0.896*** ((0.026))</td>
<td>0.087*** ((0.029))</td>
<td>85.010*** ((24.710))</td>
<td>1.621*** ((0.155))</td>
<td>15.0</td>
<td>1,169.9</td>
<td>0.978*** ((0.009))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

***Significant at 0.01 level. **Significant at 0.05 level. *Significant at 0.10 level. The regression and GED parameters are two-tailed tests. The LM-ARCH is a one-tail test.
Table 9: Correlation Matrix of Rolling Estimates of Correlations between the Conditional Moments of Bond Excess Returns

The following are correlations between rolling estimates of correlations between the fitted values of the conditional mean and conditional variance of excess returns on bonds of different maturities. The 17 month rolling correlation for each bond maturity is between the conditional excess return and conditional variance as shown in Figure 1. The model used to estimate the conditional excess returns and variances is shown on Table 7 for each maturity. Using all of the time series from January 1961 to December 2002, the correlation coefficients begin in May 1962 and end in December 2002.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>τ≈3</th>
<th>0&lt;τ≤12</th>
<th>12&lt;τ≤24</th>
<th>24&lt;τ≤36</th>
<th>36&lt;τ≤48</th>
<th>48&lt;τ≤60</th>
<th>τ≈120</th>
<th>τ≈240</th>
</tr>
</thead>
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<tr>
<td>τ≈3</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0&lt;τ≤12</td>
<td>0.52</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>12&lt;τ≤24</td>
<td>0.25</td>
<td>0.80</td>
<td>1.00</td>
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<tr>
<td>24&lt;τ≤36</td>
<td>0.20</td>
<td>0.70</td>
<td>0.91</td>
<td>1.00</td>
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<tr>
<td>36&lt;τ≤48</td>
<td>0.18</td>
<td>0.62</td>
<td>0.85</td>
<td>0.97</td>
<td>1.00</td>
<td></td>
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<tr>
<td>48&lt;τ≤60</td>
<td>0.11</td>
<td>0.54</td>
<td>0.83</td>
<td>0.93</td>
<td>0.95</td>
<td>1.00</td>
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<tr>
<td>τ≈120</td>
<td>0.14</td>
<td>0.29</td>
<td>0.54</td>
<td>0.60</td>
<td>0.66</td>
<td>0.71</td>
<td>1.00</td>
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</tr>
<tr>
<td>τ≈240</td>
<td>0.14</td>
<td>0.25</td>
<td>0.42</td>
<td>0.49</td>
<td>0.50</td>
<td>0.55</td>
<td>0.56</td>
<td>1.00</td>
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</table>
Figure 1. Rolling Estimates of Correlations between the Conditional Moments of Bond Excess Returns

The graphs below plot the 17 month rolling estimates of the correlation between the fitted values of the conditional mean excess return and conditional variance for each bond maturity. The models used to estimate the excess returns and variances are reported in Table 7. Using all of the time series from January 1961 to December 2002, the correlation coefficients begin in May 1962 and end in December 2002. Shaded areas represent business cycle contractions as defined by the National Bureau of Economic Research with the beginning month defined as a peak and the ending month defined as a trough month. Non-shaded areas are business cycle expansions.
The graphs below plot the 17 month rolling estimates of the correlation between the fitted values of the conditional mean excess return and conditional variance for each bond maturity. The models used to estimate the excess returns and variances are reported in Table 7. Using all of the time series from January 1961 to December 2002, the correlation coefficients begin in May 1962 and end in December 2002. Shaded areas represent business cycle contractions as defined by the National Bureau of Economic Research with the beginning month defined as a peak and the ending month defined as a trough month. Non-shaded areas are business cycle expansions.
The graphs below plot the 17 month rolling estimates of the correlation between the fitted values of the conditional excess return and conditional variance. The model used to estimate the excess returns and variances is shown on Table 7 for each maturity. Using all of the time series from January 1961 to December 2002, the correlation coefficients begin in May 1962 and end in December 2002. Shaded areas represent business cycle contractions as defined by the National Bureau of Economic Research with the beginning month defined as a peak and the ending month defined as a trough month. Non-shaded areas are business cycle expansions.